Solutions to Problem 1.

a. Let λ_0 be the arrival rate for urgent patients. Let $Y_{0,t}$ be the number of urgent patients that arrive by time t. By the decomposition property, $Y_{0,t}$ follows a Poisson process with arrival rate $\lambda_0 = 0.14(2) = 0.28$ patients per hour.

$$\begin{aligned}\Pr\{Y_{0,12} > 6\} &= 1 - \Pr\{Y_{0,12} \le 6\} \\ &= 1 - \sum_{k=0}^{6} \frac{e^{-0.28(12)}(0.28(12))^k}{k!} \\ &\approx 0.055\end{aligned}$$

b. Let λ_2 be the overall arrival rate. Let $Y_{2,t}$ be the number of patients overall that arrive by time t. By the superposition property, $Y_{2,t}$ follows a Poisson process with arrival rate $\lambda_2 = 2 + 4 = 6$ patients per hour.

$$\begin{aligned}\Pr\{Y_{2,6} > 30\} &= 1 - \Pr\{Y_{2,6} \le 30\} \\ &= 1 - \sum_{k=0}^{30} \frac{e^{-6(6)}(6(6))^k}{k!} \\ &\approx 0.819\end{aligned}$$

Solutions to Problem 2. Let $\lambda_0 = 8$ surges per hour, $\lambda_1 = 1/18$ surges per hour, and $\lambda_2 = 1/46$ surges per hour. In addition, let $p_1 = 0.005$, and $p_2 = 0.08$.

a. Let Y_t be the number of all surges by time t. By the superposition property, Y_t follows a Poisson process with rate $\lambda = \lambda_0 + \lambda_1 + \lambda_2 \approx 8.077$ surges per hour. Therefore, $E[Y_8] \approx \lambda \cdot 8 \approx 64.6$.

b. Let $Y_{10,t}$ be the number of small surges that are computer-damaging by time t, and let $Y_{20,t}$ be the number of moderate surges that are computer-damaging by time t. By the decomposition property, $Y_{10,t}$ and $Y_{20,t}$ follow Poisson processes with rates $\lambda_{10} = p_1\lambda_1 = 1/3600$ and $\lambda_{20} = p_2\lambda_2 = 1/575$ surges per hour, respectively.

   Let $Y_{3,t}$ be the number of computer-damaging surges by time t. By the superposition property, $Y_{3,t}$ is a Poisson process with rate $\lambda_3 = \lambda_{10} + \lambda_{20} \approx 0.0020$. Therefore, $E[Y_{3,8}] = \lambda_3 \cdot 8 \approx 0.016$.

c. $\Pr\{Y_{3,8} = 0\} = \dfrac{e^{-\lambda_3(8)}(\lambda_3(8))^0}{0!} \approx 0.98$

Solutions to Problem 3.

a. Let $\{Y_{0,t} : t \ge 0\}$ be a Poisson process with rate $\lambda_0 = 400$, representing the arrival of requests for the chatbot. Let $\{Y_{1,t} : t \ge 0\}$ be a Poisson process with rate $\lambda_1 = 1000$, representing the arrival of requests for the image generator. Therefore, $Y_t = Y_{0,t} + Y_{1,t}$ follows a Poisson process with rate $\lambda = \lambda_0 + \lambda_1 = 1400$.

$$\Pr\{Y_{1.5} > 2000\} = 1 - \Pr\{Y_{1.5} \le 2000\} = 1 - \sum_{k=0}^{2000} \frac{e^{-1400(1.5)}(1400(1.5))^k}{k!} \approx 0.985$$

b. Let $Y_{A,t}$ and $Y_{B,t}$ be the number of arrivals to servers A and B, respectively. By the decomposition property, $Y_{A,t}$ and $Y_{B,t}$ follow a Poisson process with rate $\lambda_A = (1/2)\lambda = 700$ and $\lambda_B = (1/2)\lambda = 700$, respectively. Moreover, these two Poisson processes are independent. Therefore,

$$\begin{aligned}\Pr\{Y_{A,1.5} > 1000 \text{ and } Y_{B,1.5} > 1000\} &= \Pr\{Y_{A,1.5} > 1000\}\Pr\{Y_{B,1.5} > 1000\} \\ &= (1 - \Pr\{Y_{A,1.5} \le 1000\})(1 - \Pr\{Y_{B,1.5} \le 1000\}) \\ &= \left(1 - \sum_{k=0}^{1000} \frac{e^{-700(1.5)}(700(1.5))^k}{k!}\right)\left(1 - \sum_{k=0}^{1000} \frac{e^{-700(1.5)}(700(1.5))^k}{k!}\right) \\ &\approx 0.877\end{aligned}$$

Solutions to Problem 4. Let $Y_{0,t}$ be the number of trucks by time t, which by the decomposition property, follows a Poisson process with rate $\lambda_0 = 0.05(1) = 0.05$. Similarly, let $Y_{1,t}$ be the number of all other automobiles by time t, which follows a Poisson process with rate $\lambda_1 = 0.95(1) = 0.95$.

a. $$\begin{aligned}\Pr\{Y_{0,60} \geq 1 &= 1 - \Pr\{Y_{0,60} = 0\} \\ &= 1 - \frac{e^{-(0.05)(60)}((0.05)(60)^k}{k!} \\ &\approx 0.95\end{aligned}$$

b. Since $\{Y_{0,t} : t \geq 0\}$ is independent of $\{Y_{1,t} : t \geq 0\}$, what happened to $Y_{0,t}$ is irrelevant. Therefore, $E[Y_{1,60}] = 60(0.95) = 57$, and the expected total number of automobiles that have passed by in that hour is $10 + 57 = 67$.

Solutions to Problem 5.

a. In this situation, time-stationary means that demand is not time-dependent (i.e., no seasonal demand), and the market for A, B, C is not time-dependent (i.e., not increasing or decreasing).

b. Let Y_t = total sales up to week t. Y_t follows a Poisson process with arrival rate $10 + 10 = 20$.

$$\begin{aligned}\Pr\{Y_1 > 30\} &= 1 - \Pr\{Y_1 \leq 30\} \\ &= 1 - \sum_{k=0}^{30} \frac{e^{-20(1)}(20(1))^k}{k!} \approx 0.013\end{aligned}$$

c. Let $Y_{A,t}$ = total sales of A up to week t, $Y_{B,t}$ = total sales of B up to week t, and $Y_{C,t}$ = total sales of C up to week t. $Y_{A,t}$ follows a Poisson process with arrival rate $20(0.2) = 4$, $Y_{B,t}$ follows a Poisson process with arrival rate $20(0.7) = 14$, and $Y_{C,t}$ follows a Poisson process with arrival rate $20(0.1) = 2$.

Expected sales in 1 month:

$$E[Y_{A,4}] = 4(4) = 16 \qquad E[Y_{B,4}] = 14(4) = 56 \qquad E[Y_{C,4}] = 2(4) = 8$$

Expected person-hours for 1 month:

$$25E[Y_{A,4}] + 15E[Y_{B,4}] + 40E[Y_{C,4}] = 25(16) + 15(56) + 40(8) = 1560$$

d. Let $Y^L_{B,t}$ = Louise's total sales of B up to week t. $Y^L_{B,t}$ follows a Poisson process with rate $10(0.7) = 7$.

$$\begin{aligned}\Pr\{Y^L_{B,2} - Y^L_{B,1} > 5 \text{ and } Y^L_{B,1} > 5\} &= \Pr\{Y^L_{B,2} - Y^L_{B,1} > 5\}\Pr\{Y^L_{B,1} > 5\} \quad \text{(independent increments)} \\ &= \Pr\{Y^L_{B,1} > 5\}\Pr\{Y^L_{B,1} > 5\} \quad \text{(stationary increments)} \\ &= \left(1 - \sum_{k=0}^{5} \frac{e^{-7(1)}(7(1))^k}{k!}\right)^2 \approx 0.4890\end{aligned}$$